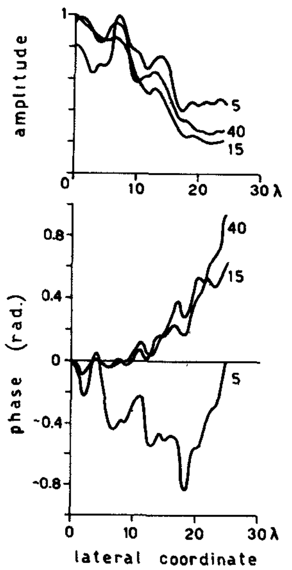

Fig. 6. Case of an impinging unlimited plane wave: normalized field amplitude and phase, plotted versus x, at several frames.

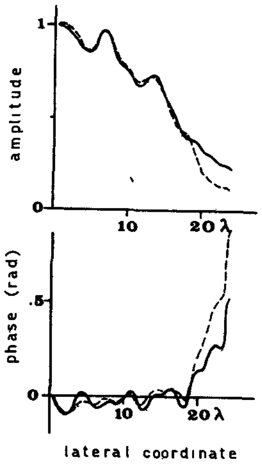

Fig. 7. The field at the 200th frame (solid lines) and the iterative field of the open-resonator theory (dashed lines) for the same case as Fig. 6.

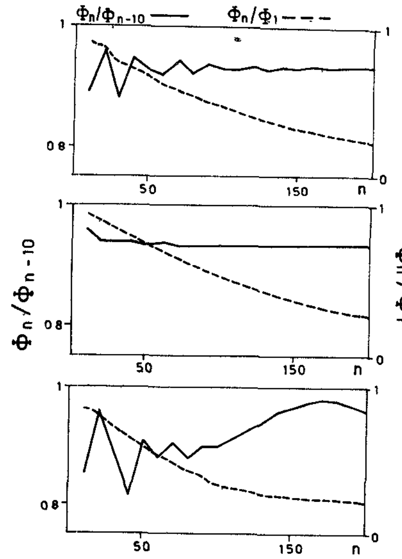

Fig. 8. The ratio Φ_n/Φ_{n-10} (solid lines) and the ratio Φ_n/Φ_1 (dashed lines), plotted versus n, when the beam waveguide is illuminated by a truncated plane wave (top), the iterative field of open resonator theory (center), and an unlimited plane wave (bottom).

plane truncated wavefront, emitted at a distance $L = 100\ \lambda$ before the first frame. Fig. 4 shows the field distribution at a number of frames, plotted versus x (for symmetry reasons, the field is plotted only for $x \geq 0$). Fig. 5 shows the quasi-stationary field distribution, which is reached after about 100 cells, compared with the iterative field distribution of the equivalent open resonator theory [6], [7].

Figs. 6 and 7 refer to an impinging plane wave. Dashed lines represent, as before, the iterative field.

Fig. 8 shows the quantities Φ_n/Φ_1 and Φ_n/Φ_{n-10}, plotted versus n, where

$$\Phi_n = \int_{-(a+l)}^{a+l} | v_n(x_n) |^2 dx_n$$

represents the power flux through the nth frame.

Our treatment of open-beam waveguides has a general character, so that it can be applied not only to periodic but also to nonperiodic sequences of optical elements. Thus it is suitable for the study of the effects of errors in the position (both in the longitudinal and in the transverse senses) of the elements, and of differences in the optical properties of them, as well as of possible bends in the axis of the beam waveguide (the planes π_i are not necessarily parallel to one another). In addition, it may be generalized so as to take into account the backscattering of the optical elements, which was neglected in (5).

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Application of the Beam Mode Expansion to the Analysis of Noise Reduction Structure

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***Abstract*—The beam mode expansion method used to discuss the diffraction problem by an aperture is applied to the analysis of the noise reduction structure consisting of two aperture stops. The incident field is a fundamental wave beam whose amplitude distribution is Gaussian. The transmitted field through the structure can be represented as a sum of beam mode functions and is regarded as a signal. The noise which is originated from the spontaneous**

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emission is added to the incident Gaussian wave beam. The signal-to-noise ratio (SNR) in the output is discussed and optimum conditions are obtained numerically.

I. INTRODUCTION

The analysis of an optical structure which includes elements of finite aperture is treated by the Fresnel integral. For complicated systems, however, the numerical integration is very difficult.

In the case of wave beam transmission, the beam mode expansion method might be convenient for the analysis of such systems.

In a previous paper [1], a system of two aperture stops is analyzed for an incident field with a Gaussian field distribution by using the beam mode expansion method and the conditions which maximize the power of the fundamental beam mode in the output are obtained. These conditions coincide with those obtained to maximize the output power of this system for a prolate spheroidal-wave distribution [2].

In some practical cases, however, the transmitted field which includes not only the fundamental beam mode but also the higher modes might be regarded as a signal. And the noise originated from the spontaneous emission of atoms is added to the signal in laser amplifiers.

In this short paper, the SNR in the output is discussed and the optimum incidence conditions which give the maximum SNR for some aperture configurations are obtained numerically. A detector collects the whole field through the system. The theory is based on the scalar paraxial approximation.

The calculations are carried out for circular geometries and the first ten higher modes are considered.

II. SIGNAL, NOISE, AND THE SNR IN THE OUTPUT

The diffraction field $U_{mn}(r,\theta,z)$ from an aperture for an incident wave beam $\psi_{mn}(r,\theta,z)$ with a Laguerre–Gaussian field distribution [3] which has the beam waist at $z = -z_s$ where its spot size is w_s, is obtained by using the Kirchhoff–Huygens formula. The spot size of a wave beam is defined by the radius at which the exponential term in the field distribution falls to e^{-1}. This field is then expanded into a sum of beam mode functions, which have the same beam parameters as the incident wave beam, as follows:

$$U_{mn}(r,\theta,z) = \sum_{\bar m,\bar n=0}^{\infty} C_{mn}^{\bar m \bar n}\psi_{\bar m \bar n}(r,\theta,z). \tag{1}$$

The expansion coefficient $C_{mn}^{\bar m \bar n}$ is, by using the orthonormality of $\{\psi_{\bar m \bar n}\}$, given by [4]

$$C_{mn}^{\bar m \bar n} = \left(\frac{n!}{(n+m)!}\right)^{1/2}\left(\frac{\bar n!}{(\bar n+m)!}\right)^{1/2} \exp\left[j(2n-2\bar n)\tan^{-1}\xi_0\right] \cdot \sum_{p=0}^{n}\sum_{q=0}^{\bar n}\frac{(-1)^{p+q}(p+q+m)!}{p!q!}\binom{n+m}{n-p}\binom{\bar n+m}{\bar n-q} \cdot\left[1-\exp(-\eta_0^2a^2)\sum_{s=0}^{p+q+m}(s!)^{-1}(\eta_0^2a^2)^s\right],$$

$$(\bar m = m), \qquad C_{mn}^{\bar m \bar n} = 0, \qquad (\bar m \neq m) \tag{2}$$

where

$$\xi_0 = \frac{2(z_0+z_s)}{kw_s^2}, \qquad \eta_0 = \frac{\sqrt{2}}{w_s(1+\xi_0^2)^{1/2}} \tag{3}$$

$k = 2\pi/\lambda$ (λ: wavelength) is the wavenumber of the field, and a is the radius of the aperture.

By using this beam mode expansion method we can easily represent the transmitted field $U(r,\theta,z)$ from the system of two aperture stops in Fig. 1 as a sum of the beam mode functions as follows:

$$U(r,\theta,z) = \sum_{\bar n=0}^{\infty}\sum_{t=0}^{\infty} C_{mn}^{(1)m\bar n}C_{m\bar n}^{(2)mt}\psi_{mt}(r,\theta,z) \tag{4}$$

where

$$C_{mn}^{(i)m\bar n} = \left(\frac{n!}{(n+m)!}\right)^{1/2}\left(\frac{\bar n!}{(\bar n+m)!}\right)^{1/2} \exp\left[j(2n-2\bar n)\tan^{-1}\xi_{i0}\right] \cdot \sum_{p=0}^{n}\sum_{q=0}^{\bar n}\frac{(-1)^{p+q}(p+q+m)!}{p!q!}\cdot\binom{n+m}{n-p}\binom{\bar n+m}{\bar n-q} \cdot\left[1-\exp(-\eta_{i0}^2a_i^2)\sum_{s=0}^{p+q+m}(s!)^{-1}(\eta_{i0}^2a_i^2)^s\right], \qquad (i=1,2) \tag{5}$$

$$\xi_{10} = \frac{2z_s}{kw_s^2}, \; \xi_{20} = \frac{2(d+z_s)}{kw_s^2}, \; \eta_{i0} = \frac{\sqrt{2}}{w_s(1+\xi_{i0})^{1/2}}, \qquad (i=1,2) \tag{6}$$

and a_1,a_2 are the radii of the apertures A_1,A_2, respectively. Therefore, the power transmission coefficient of the system which is the ratio of the transmitted power S to the incident power S_0 for the fundamental mode ($m = n = 0$) incidence with a Gaussian transverse field distribution is given by

$$\tau = \sum_{t=0}^{\infty}\left|\sum_{\bar n=0}^{\infty} C_{00}^{(1)0\bar n}C_{0\bar n}^{(2)0t}\right|^2. \tag{7}$$

The transmitted power S is, in this short paper, regarded as a signal.

The transmitted noise through the system is given by [2]

$$N = \mathcal{A}N_0 \tag{8}$$

$$\mathcal{A} = \frac{\pi^2a_1^2a_2^2}{d^2\lambda^2} \tag{9}$$

where N_0 is the amount of "noise per mode," that is, the amount of noise radiated into a solid angle, and $a_1/d \ll 1, a_2/d \ll 1$ are assumed.

The parameter $\mathcal{A}$ is called the acceptance factor of the system. In obtaining the transmitted noise, a polarizer is used to reduce it by a factor of two. The polarizer has no effect on the signal whose polarization is linear.

The SNR in the output is, therefore, given by

$$\frac{S}{N} = \frac{\tau}{\mathcal{A}}\frac{S_0}{N_0}. \tag{10}$$

The ratio S_0/N_0 is the ideal value of the SNR.

In the next section, the numerical computations of $\tau/\mathcal{A}$ are carried

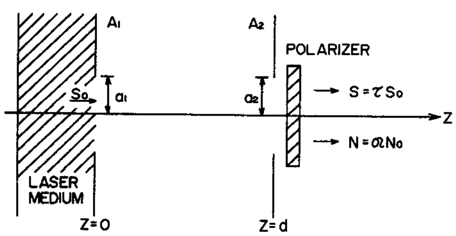

Fig. 1. The noise reduction structure which consists of two aperture stops. The acceptance factor $\mathcal{A}$ of this structure is given by $\mathcal{A} = \pi^2a_1^2a_2^2/d^2\lambda^2$.

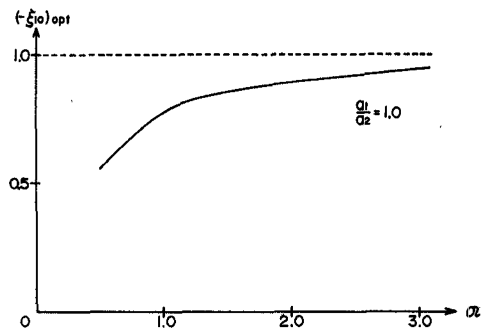

Fig. 2. The optimum position of the beam waist of the incident wave beam. The dotted line shows the result obtained in [1] and [2].

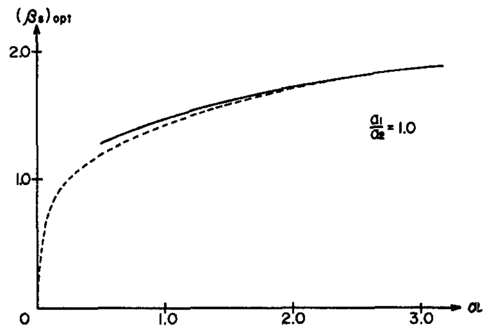

Fig. 3. The optimum spot size of the incident wave beam. The dotted line shows the result obtained in [1] and [2].

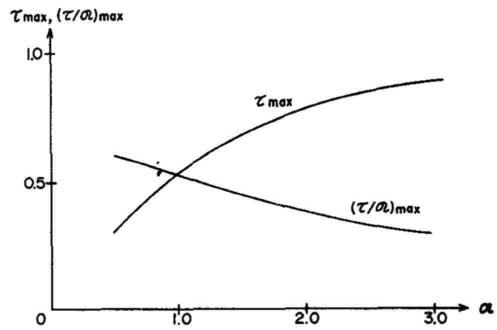

Fig. 4. The maximum transmitted power τ_{max} and the maximum ratio $(\tau/\mathcal{A})_{max}$.

out and the optimum incidence conditions are obtained for given aperture configurations.

III. NUMERICAL COMPUTATIONS

In calculating the ratio $\tau/\mathcal{A}$, we use the first ten higher modes in the series of (7). Increasing the number of mode has little effect on the results except when the acceptance factor $\mathcal{A}$ is very small.

Figs. 2 and 3 show the optimum incidence conditions which maximize the ratio $\tau/\mathcal{A}$ for $a_1/a_2 = 1.0$. The parameter β_s is given by $\beta_s = (a_1a_2)^{1/2}/w_s$. The dotted lines in these figures show the optimum conditions obtained in [1] and [2].

From these figures, the optimum position of the beam waist $(-z_s)$ and the smallest spot size (w_s) of the incident wave beam are obtained as follows:

$$\begin{aligned}(w_s/d)_{opt} &= (2/kd)^{1/2}\mathcal{A}^{1/4}/(\beta_s)_{opt}\\ (-z_s/d)_{opt} &= (kd/2)\,(w_s/d)^2_{opt}(-\xi_{10})_{opt}.\end{aligned} \tag{11}$$

Comparing the results obtained here with those in [1] and [2], we can see that the optimum position of the beam waist $(-z_s/d)_{opt}$ is closer to the first aperture A_1 than that in [1] and [2], and the difference increases as $\mathcal{A}$ becomes smaller. There is no such salient difference in the case of the optimum spot size. This tendency holds true for $a_1 \neq a_2$.

The maximum transmitted power and the ratio $\tau/\mathcal{A}$ are shown in Fig. 4. The most noteworthy point in Fig. 4 is that the maximum transmitted power and the ratio $\tau/\mathcal{A}$ are independent of the ratio a_1/a_2 and depend only upon the acceptance factor $\mathcal{A}$.

In the figures presented here, the results for small values of $\mathcal{A}$ are not shown. To obtain them, we must take into consideration more higher modes. But when $\mathcal{A}$ is very small, the transmitted signal power is also very small. Therefore, systems with small $\mathcal{A}$ may have little practical importance.

IV. CONCLUSIONS

The noise reduction structure consisting of two aperture stops in laser amplifiers is analyzed by using the beam mode expansion method.

From the numerical results, we can see that the optimum position of the beam waist of the incident wave beam is closer to the input aperture than that obtained by Kogelnik and Yariv [2]. The difference between them increases as the acceptance factor becomes smaller.

The maximum SNR in the output depends only upon the acceptance factor. The smaller the acceptance factor, the better the SNR.

This short paper gives one of the examples which show the usefulness of the beam mode expansion method. More complex systems, such as those which include lenses, can be treated in the same way.

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